MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/2022

TMA 1301 – COMPUTATIONAL METHODS

(All sections / Groups)

5 OCTOBER 2022
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **4 pages** only with **4 Questions.**
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each solution of a question on a new page.**
4. Show all steps.

Question 1 [10marks]

Consider the function $f(x) = e^x - 4x$ on interval [0, 0.5] under ***Bisection method***. Use FOUR decimal places for all workings.

(a) State the three ***stopping criteria*** for the iteration of Bisection method. [1.5 marks]

(b) Find the number of required iterations to fulfil the tolerance 7×10^{-2}. [2.5 marks]

(c) Based on the number of iterations obtained in (b), find the approximated root by copying and completing the following table. [3 marks]

n	a_n	b_n	p_n	$f(a_n)$	$f(p_n)$	$f(a_n)f(p_n)$

(d) Assume that the actual root is 0.3574, find the absolute error and relative error by using the approximated root obtained in (c). [3 marks]

Question 2 [10 marks]

Consider the definite integral $\int_1^2 e^{2x} dx$. Use FOUR decimal places for all workings.

(a) Find the actual value of the definite integral. [2 marks]

(b) Find the number of subintervals if Composite Trapezoidal Rule is used to approximate the definite integral with an error of at most 8×10^{-1}. [5 marks]

[HINT: Error formula = $-\frac{b-a}{12} h^2 f''(\xi)$ for $a < \xi < b$]

(c) With the number of subintervals obtained in (b), find the approximated value for the definite integral using the Composite Trapezoidal Rule. [3 marks]

Continued.....

Question 3 [10 marks]

(a) Given the matrix $B = \begin{bmatrix} 5 & -4 \\ -8 & 2 \end{bmatrix}$, find the following.

(i) Show that the inverse of matrix B^{-1} exists. [1 mark]

(ii) Compute B^{-1} by using the answer obtained in (i). Display your answers in the simplest fraction. [1.5 marks]

(b) Consider the following system of linear equations:

$$\begin{array}{rcrcrcr} 2x_1 & + & x_2 & - & x_3 & = & 1 \\ -4x_1 & + & 2x_2 & + & 4x_3 & = & 0 \\ 6x_1 & + & 3x_2 & - & 2x_3 & = & -5 \end{array}$$

(i) Convert the system of linear equations above into a matrix equation, $AX = B$. [0.5 mark]

(ii) Find an LU factorization of the matrix A obtained in (i). [3 marks]

(iii) Solve the system of linear equations above using the LU factorization obtained in (ii). [4 marks]

Continued.....

Question 4 [10 marks]

Consider the following data of the form $y = ax^b$:

x_i	1	2	3	4	5
y_i	0.5	1.7	3.4	5.7	8.4

(a) The exponential function $y = ax^b$ can be approximated to the form of linear least squares $Y = A + bX$. Find the expression of Y, A and X. [3 marks]

(b) With the answers obtained in (a), fill up the following table. Show all answers in FOUR decimal places. [4 marks]

HINT:

$$a = \frac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}, \quad b = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

x	y	X	Y	X^2	Y^2
1	0.5				
2	1.7				
3	3.4				
4	5.7				
5	8.4				
Total					

(c) Using the table obtained in (b), find the values of a and b for $y = ax^b$. [2 marks]

(d) Using the equation $y = ax^b$ obtained in (c), find the value of y when $x = 3.8$. [1 mark]

End of Exam Questions.